Problem 2:

1. (Smoke → Fire) → (¬Smoke → ¬Fire)

(¬Smoke v Fire) → (Smoke v ¬Fire) implication elimination

¬(¬Smoke v Fire) v (Smoke v ¬Fire) implication elimination

(¬ (¬Smoke) ^ ¬Fire)) v (Smoke v ¬Fire) de Morgan

(Smoke ^ ¬Fire) v (Smoke v ¬Fire) double-negation elimination

(Smoke ^ ¬Fire) v Smoke v ¬Fire associativity of v

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Smoke | Fire | ¬Smoke | ¬Fire | (Smoke → Fire) | (¬Smoke → ¬Fire) | entire |
| 0 | 0 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 0 | 1 | 0 | 0 |
| 1 | 0 | 0 | 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 0 | 1 | 1 | 1 |

Neither

1. (Smoke → Fire) → ((Smoke v Heat) → Fire)

(¬Smoke v Fire) → (¬(Smoke v Heat) v Fire) implication elimination

¬(¬Smoke v Fire) v (¬(Smoke v Heat) v Fire) implication elimination

(Smoke ^ ¬Fire) v ((¬Smoke ^ ¬Heat) v Fire) de Morgan

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Smoke | Heat | Fire | Smoke ^ ¬Fire | ((¬Smoke ^ ¬Heat) v Fire) | entire |
| 0 | 0 | 0 | 0 | 1 | 1 |
| 0 | 0 | 1 | 0 | 1 | 1 |
| 0 | 1 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 0 | 1 | 1 |
| 1 | 0 | 0 | 1 | 0 | 1 |
| 1 | 0 | 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |

Neither

(c) ((Smoke ^ Heat) → Fire) ↔ ((Smoke → Fire) v (Heat → Fire))

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Smoke | Heat | Fire | (Smoke  ^ Heat) | ((Smoke ^  Heat) → Fire) | (Smoke  → Fire) | (Heat  → Fire) | ((Smoke → Fire)  v (Heat → Fire)) | entire |
| 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 |
| 0 | 0 | 1 | 0 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 0 | 0 | 1 | 1 | 0 | 1 | 1 |
| 0 | 1 | 1 | 0 | 1 | 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 | 1 | 0 | 1 | 1 | 1 |
| 1 | 0 | 1 | 0 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |

Valid